the incident vortex has its maximum tangential velocity at the radius of the core. At this instant, the incident vortex not only distorts but also splits into upper and lower vortices. The pair of shed vortices roll up into a strong vortex of circular shape. At $t = 1.5$, the split incident vortices interact with the flow in the boundary layer and are convected at different local velocities. The shed vortex is convected at a local velocity, which is much slower than that of the lower split incident vortex as shown in the figure. This occurs because the rotation of the shed vortex is opposite to that of the incident vortex. No additional vortex is shed near the leading edge as seen in the figure. This is because the local angle of attack becomes small.

The vortex shedding process at $t = 0.6$ and $t = 0.9$ is shown in detail in Fig. 2 , which clearly indicates that the vortex pair rolls up into a single circular vortex at $t = 0.9$.

Comparisions between the experimental results of Ref. 2 and the present computed ones are not included here because of the different structures and strengths of the incident vortices studied in the experiment and the present analysis. Nevertheless, the predicted distortion and splitting of the incident vortex, and vortex shedding near the leading edge have behaviors similar to those observed in the experiment.[2]

Conclusions

Vortex-wedge interaction is investigated using the hybrid, vortex sheet-random vortex method, including viscous effects, and a fast vortex method, involving a large number of vorticity particles. The Schwarz-Christoffel transformation is applied to generate the grid system around the wedge. The shed vortex is formed by the vorticity particles generated all along the surface. Distortion of the incident vortex during the interaction and the appearance of a secondary shed vortex near the leading edge are clearly presented. A pair of vortices are shed and they roll up into a single circular vortex. It is presumed that a variety of shed vortex patterns may prevail depending upon the incident vortex size, strength, and initial position.

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Determination of Drag of a Circular Cylinder

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Introduction

MEASUREMENTS in the wake indicate that the minimum downstream distance behind a smooth circular cylinder, beyond which the contribution from the Reynolds normal stresses to the momentum integral is negligible, is equal to 30 diameters.

The drag coefficient C_D of a circular cylinder is given by

$$C_D = \frac{-2F}{U_1^2 d} \tag{1}$$

where F is the kinematic force per unit length of the cylinder given by[1]

$$F = \int_{-\infty}^{\infty} \left[U(U - U_1) + (\overline{u^2} - \overline{v^2})\right] \mathrm{d}y \tag{2}$$

where U is the local mean velocity; U_1 is the freestream velocity, assumed to remain constant in the x direction; u and v are longitudinal and lateral velocity fluctuations; and d is the cylinder diameter. C_D can be written as the sum of two integrals:

$$C_D = \underbrace{2\int_{-\infty}^{\infty} \frac{U}{U_1}\left(\frac{U_1 - U}{U_1}\right) \mathrm{d}\left(\frac{y}{d}\right)}_{I_1} + \underbrace{2\int_{-\infty}^{\infty} \left(\frac{\overline{v^2} - \overline{u^2}}{U_1^2}\right) \mathrm{d}\left(\frac{y}{d}\right)}_{I_2} \tag{3}$$

At a sufficiently large distance from the body, I_2 can be neglected and the drag coefficient reduces to

$$C_D = 2\int_{-\infty}^{\infty} \frac{U}{U_1}\left(\frac{U_1 - U}{U_1}\right) \mathrm{d}\left(\frac{y}{d}\right) = \frac{2\theta}{d} \tag{4}$$

where θ is the momentum thickness of the wake.[2] Equation (4) indicates that the drag coefficient can be determined from only a profile of mean velocity across the wake. Practical considerations[3] require that the velocity profile is measured at relatively small values of x/d. In this region, the contribution from the Reynolds normal stresses (I_2) is not necessarily negligible and may result in significant errors in the value of C_D. In earlier methods,[4,5] a correction for the difference in the mean static pressure between the freestream and the wake was included in Eq. (4). This correction arises implicitly from the y-momentum equation

$$\frac{\partial \overline{v^2}}{\partial y} = -\frac{\partial \overline{p}}{\partial y}$$

Integration of this yields

$$\overline{v^2} + \overline{p} = p_1 \tag{5}$$

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Table 1 Values of I_1, I_2, and C_D [a]

	x/d	I_1	I_2	C_D
Present experiment $d = 12.5$ mm $R_d \approx 5600$	5	0.671	0.189	0.860
	10	0.728	0.122	0.850
	20	0.821	0.043	0.864
	30	0.891	0.005	0.896
	40	0.889	−0.007	0.882
	50	0.907	−0.009	0.898
	60	0.898	−0.010	0.888
Cantwell and Coles[b]: $R_d = 1200$				0.960
$R_d = 2000$				0.820
$R_d = 5000$				0.910
Data of Browne et al.[1] $d = 2.67$ mm $R_d \approx 1200$	420	0.973	−0.003	0.97

[a] Errors in the estimate of I_1 and I_2 are of the order ±2%.
[b] Values of C_D were obtained by drawing a mean curve through the data of several investigators (Fig. 1 of Cantwell and Coles[8]).

where p_1 is the kinematic freestream static pressure. The use of Eq. (5) to determine the correct value of C_D does not account for the contribution of $\overline{u^2}$ which may not be small. Taylor[6] pointed out that the error in the expression for C_D, obtained by Betz[4] and Jones[5] via a control-volume analysis without explicitly accounting for the normal stresses, may be small. The results presented in the next section indicate that the contribution from the normal stresses can be significant at small x/d.

Measurements close to the cylinder indicate that $\overline{v^2} > \overline{u^2}$, possibly due to the vortex shedding and the relatively intense crossflow mixing associated with the motion induced by the vortices. As x/d increases, both $\overline{u^2}$ and $\overline{v^2}$ decrease, but in the far wake $\overline{u^2}$ becomes larger than $\overline{v^2}$. This indicates that at some downstream location, the contribution from the normal stresses is zero (i.e., $I_2 = 0$) and an accurate value of C_D can be obtained from only a mean velocity profile at that location. Our aim was also to determine this location and estimate C_D as accurately as possible.

Results

Measurements were made in the wake of a cylinder, over the range $0 < x/d < 60$, of diameter $d = 12.5$ mm. A constant freestream velocity U_1 of 6.7 m/s was used and the corresponding Reynolds number $R_d = U_1 d/\nu$ was 5600. At each x/d location, profiles of $\overline{U}$, $\overline{u^2}$, and $\overline{v^2}$ were determined by traversing an X-probe across the wake. The integrals I_1 and I_2 were estimated from these profiles. Table 1 summarizes the results obtained in the present experiments. At $x/d = 30$, I_2 is nearly zero, indicating that the contribution to C_D from the normal stresses at this location is negligible. For $x/d > 30$, I_2 changes sign but its magnitude is quite small. At $x/d = 5$, I_2 contributes 22% to C_D and the contribution decreases as x/d increases. At $x/d = 20$, I_2 contributes only 5% to the value of C_D. For $x/d > 30$, the contribution of I_2 is negative, but negligible. A value of C_D, estimated for the far wake data of Browne et al.[7] is also shown in Table 1.

Values of C_D given in Table 1 are in reasonable agreement with available results in the literature (e.g., Cantwell and Coles[8]). Table 1 also indicates that for $x/d \geq 30$, C_D is nearly constant. Allowing for the experimental uncertainty, the minimum distance at which the mean velocity profile can be used to determine C_D without correcting for the Reynolds-normal-stress term appears to be $30d$. This distance should be sufficiently large to allow any local freestream pressure disturbance, arising from the insertion of the cylinder in the (finite area) working section, to disappear. The minimum distance may depend on initial conditions (such as the nature of the cylinder surface or the freestream turbulence) and the Reynolds number. It should be finally noted that the present estimate for this distance applies strictly to the flow behind a circular cylinder. When the wake-generating body is streamlined, or partially streamlined, the available u^2 and v^2 data, e.g., the measurements of Chevray and Kovasznay[9] for the wake of a thin flat plate or Chevray[10] for the wake of a six-to-one spheroid suggest that I_2 does not change sign in these flows.

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Prediction of Inviscid Stagnation Pressure Losses in Supersonic Inlet Flows

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TO achieve a desired mass flow rate through a high-speed inlet, the stagnation pressure must be above some critical value. Stagnation, or total, pressure losses in bounded supersonic flows, such as those in a supersonic inlet, can result in the flow becoming unchoked (i.e., subsonic). It is important, therefore, to be able to predict these pressure drops if one is to produce a viable inlet design. The primary intention of this Note is to quantify the stagnation pressure losses associated with shock-wave systems that may be present in such high Mach number flows.

Figure 1, from Goldberg and Hefner,[1] is a graph of the maximum contraction ratio that can be achieved by a two-dimensional inlet and still be able to pass the intercepted mass

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